A Pythagorean tree fractal shape stub-loaded resonator as a UWB bandpass filter with wide stopband

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Abstract

A compact ultrawideband (UWB) bandpass filter (BPF) is designed by harnessing the efficacy of a Pythagorean tree fractal shape stub-loaded resonator. The design inherently provides the passband transmission poles, which make it convenient to be used in wide passband filtering circuits. The number and the position of the resonating modes can be controlled by increasing the iterations of the Pythagorean tree, as analyzed using odd- and even-mode analysis. Design steps of the BPF are detailed. The designed UWB BPF takes up a small circuit area of (12.13 × 9.59) mm². The proposed design is fabricated and measured to verify the simulated results. The stopband is extended up to 17.5 GHz with a maximum attenuation of 15 dB.

Introduction

With the increasing trends of device miniaturization and to satisfy the demand of ultrawideband (UWB) technology, several novel techniques have been developed to design compact, high selective filter with good passband and stopband characteristics. Wireless communication always demands improvement in receiver front-end circuits that play a major role in segregating the desired signals from the unwanted signal [1]. The UWB bandpass filter (BPF) is a component that has a major assistive role in enhancing the performance of other receiver active components such as LNA. Conventional filter design techniques are incompetent to satisfy the necessities of UWB BPF and it also enhances the circuit dimension. Hence, different topologies such as stub-loaded multiple-mode resonator (MMR) [2], Cascaded highpass and lowpass/bandstop filter [3], transition-based structure (microstrip/CPW or microstrip/slotline) [4], different shapes of the resonator are accepted for the UWB BPF design. With the assignment of unlicensed use of the UWB spectrum (3.1 GHz–10.6 GHz) in 2002 [5], the application of UWB systems is very much commercialized. However, to fulfil all these requirements, there is a need for miniaturization. Space-filling property and self-similarity are the two major features of fractal geometry which make it suitable to opt as one of the techniques to design UWB BPF [6]. A novel and very compact UWB BPF comprises of a fractal-shaped resonator and a composite right/left-handed transmission line is reported in [7]. Fractal-shaped UWB BPF is designed with Koch island-shaped stepped impedance microstrip line and a ground slot is made by etching a Koch fractal-shaped slot. A stub-loaded MMR-based UWB BPF is designed in [8]. The proposed MMR comprises of uniform impedance resonator (UIR) which is centrally loaded with a stepped-impedance stub and two uniform-impedance stubs at the symmetrical positions. Three even-modes and two odd-modes are allocated in the required band and both the transmission zeros (TZs) are created by the stepped-impedance stub which in turn improves the selectivity of the filter. UWB BPF with the dual stub-loaded resonator is reported in [9]. The desired resonant modes are assigned in the UWB range by simply tuning the lengths of short and open-circuited stubs. The presented filter has passband bandwidths that range from 2.9 GHz to 10.9 GHz. An MMR technique is presented in [10], which comprises three pairs of parallel coupled-line sections and a pair of open and short-circuited stubs. Two even-modes and two odd-modes are generated in the required UWB band. The skirt selectivity is good because of TZs created at both the cut-off frequencies. The compact quintuple-mode UWB BPF is reported in [11] with five resonant modes and two TZs are created at both the cut-off frequencies to increase filter’s selectivity. Radial stubs [12] loaded multi-mode resonators have been used to design UWB BPF [13, 14]. In this paper, the Pythagoras fractal tree [15] is used as a stub to create a novel multimode resonator. The structure provides a higher number of transmission poles to achieve the passband without increasing the design complexity by the inclusion of a number of assistive resonators coupled together to achieve the wide passband. From the analysis, it is observed that this topology has better miniaturization capability as compared to other fractal geometries because of its self-similar and space-filling characteristics.
The order of iteration is the key factor to improve the resonance of the proposed resonator. The filter is designed on an FR4 substrate of height 1.0 mm having a relative permittivity of 4.4. For the purpose of EM simulation, the CST Microwave Studio™ is used.

**Design of fractal tree stub-loaded MMR**

The basic unit of this fractal geometry is a square. The name of the fractal geometry reveals that the geometry is inspired by the Pythagorean Theorem [14]. The construction of this fractal geometry starts with a square of area \((S \times S)\). The larger and two smaller squares are oriented as illustrated in Fig. 1(a). The angle subtended by the two smaller squares at the point of contact is 90°. In the next iteration, the side of the first iteration is scaled down by a factor of \(\left(\frac{1}{2}\right) \times \sqrt{2}\), which adheres to the original square. At first, a wide BPF based on the fractal tree stub multimode resonator is proposed. Then the theory is developed to justify the dependence of the resonant modes of the proposed MMR on the geometrical parameters of the fractal stub-loaded MMR.

**Even–odd mode analysis**

The basic building block of the proposed MMR is a fractal tree-shaped stub loaded on to a UIR. The equivalent transmission line model of the proposed MMR is shown in Fig. 2(a) and it is analyzed using even–odd mode method. The even-mode circuit is developed by adding a magnetic wall along the symmetrical plane (TT') as shown in Fig. 2(b). Whereas, the odd-mode circuit can be achieved by adding an electrical wall along the symmetrical plane TT' as shown in Fig. 2(c). For even-mode excitation, the resulting input impedance can be expressed as

\[
\begin{align*}
Z_{in,e1} & = \frac{Z_6 \cdot (2 \cdot j / Z_7) \cdot \tan \theta_1 - j \cdot Z_6 \cdot \tan \theta_0}{Z_6 + j \cdot (2 \cdot j / Z_7) \cdot \tan \theta_1 \cdot \tan \theta_0}, \\
Z_{in,e2} & = \frac{Z_5 \cdot (2 \cdot j \cdot Z_{in,e1}) + j \cdot Z_5 \cdot \tan \theta_3}{Z_5 + j \cdot (2 \cdot j \cdot Z_{in,e1}) \cdot \tan \theta_3}, \\
Z_{in,e3} & = \frac{Z_4 \cdot (2 \cdot j \cdot Z_{in,e2}) + j \cdot Z_4 \cdot \tan \theta_4}{Z_4 + j \cdot (2 \cdot j \cdot Z_{in,e2}) \cdot \tan \theta_4}, \\
Z_{in,e4} & = \frac{Z_3 \cdot (2 \cdot j \cdot Z_{in,e3}) + j \cdot Z_3 \cdot \tan \theta_5}{Z_3 + j \cdot (2 \cdot j \cdot Z_{in,e3}) \cdot \tan \theta_5}, \\
Z_{in,e5} & = \frac{Z_{in,e4} \cdot \frac{Z_2}{2} \cdot \tan \theta_3}{Z_{in,e4} \cdot j \cdot (Z_2/2) \cdot \tan \theta_3 + j \cdot (Z_{in,e4}) \cdot \tan \theta_3},
\end{align*}
\]

Therefore,

\[
Y_{in,even} = \frac{1}{Z_1} \left[ \frac{Z_1 + j \cdot Z_{in,e5} \cdot \tan(\theta_3/2)}{Z_{in,e5} + j \cdot Z_1 \cdot \tan(\theta_3/2)} \right].
\]
For odd-mode excitation, the resulting input impedance can be expressed as

\[ Y_{in,odd} = j \cdot Y_1 \cdot \cot (\theta_1 / 2). \]  

(2)

By equating final expression (1) and (2) to zero; the resonating modes can be obtained. The expressions are analyzed in MATLAB™ and for the resonating frequencies of 3.3 GHz, 6.4 GHz, and 9.5 GHz, different impedances and electrical lengths are obtained as \( Z_1 = 113.49 \Omega, \ Z_2 = 40.97 \Omega, \ Z_3 = 50.47 \Omega, \ Z_4 = 61.36 \Omega, \ Z_5 = 72.99 \Omega, \ Z_6 = 85.03 \Omega, \ Z_7 = 97.33 \Omega, \ \theta_1 = 170.87^\circ, \ \theta_2 = 41.58^\circ, \ \theta_3 = 29.36^\circ, \ \theta_4 = 20.46^\circ, \ \theta_5 = 14.28^\circ, \ \theta_6 = 9.99^\circ, \) and \( \theta_7 = 6.98^\circ. \) One TZ is also acquired at 2.8 GHz.

The performance of the Pythagorean tree-based MMR is further investigated to comprehend the effect of the fractal iterations. The transmission coefficient is plotted in Fig. 3 under the weak coupling condition of the proposed resonator. It can be seen that the existence of the first resonating mode depends on the order of the iteration. As the order of the iteration increases, the first resonating mode becomes more prominent. The dimensions are chosen such that the first resonating mode appears at the lower cut-off of the UWB band. The second resonating mode has very negligible dependence on iteration, whereas the location of the third mode entirely depends on the iteration order. At least three resonating modes are required in the passband to implement UWB BPF. So the fifth iteration Pythagorean tree is suitable for constructing a UWB filter as it produces the required number of resonating modes in the desired band. The EM-simulation response shows that the desired modes are placed at 3.37 GHz, 6.39 GHz, and 9.47 GHz. These values are close to the analytically synthesized values. A TZ is also observed near to the lower cut-off frequency as claimed earlier.

Proposed UWB BPF and results

The final layout of the UWB BPF is shown in Fig. 4. The interdigital coupled line is used as a feeding network to ensure strong EM coupling. Length of the feed line is chosen as half wavelength.
long at the center frequency (6.85 GHz) of the UWB passband. Final dimensions of the proposed filter are: \( l_u = 11.6 \text{ mm} \), \( w_u = 0.3 \text{ mm} \), \( l_o = 4.3 \text{ mm} \), \( w_o = 0.26 \text{ mm} \), \( g = 0.13 \text{ mm} \), \( s = 2.62 \text{ mm} \), \( s_1 = 1.9 \text{ mm} \), \( s_2 = 1.33 \text{ mm} \), \( s_3 = 0.94 \text{ mm} \), \( s_4 = 0.66 \text{ mm} \), and \( s_5 = 0.46 \text{ mm} \). The surface current distribution at transmission poles is illustrated in Fig. 5 for further verification. It reveals that the high current is induced at the resonating frequencies which validate the resonance behavior as claimed earlier.

The photograph of the proposed UWB BPF is shown in Fig. 6. All the measurements are done using Rhode and Schwarz ZVA 40 VNA. The measured results agree well with the simulated results as shown in Fig. 7. The measured result shows suitable passband performance with a wide stopband extended up to 17.5 GHz. Two TZs at 2.5 and 11.8 GHz improves the selectivity of the UWB BPF. The TZ near to the upper cut-off frequency is placed due to the interdigital coupled line. The proposed filter covers the whole band of UWB with maximum \( |S_{21}| \) of -1.5 dB. The value of \( |S_{11}| \) dB is better than -10 dB within the passband. Figure 8 reflects that the measured group delay response is flat in the passband which suggests distortion-free transmission. A comparative study is presented in Table 1, which highlights the better performance of the proposed technique by considering the previously reported works. It can be observed that the proposed filter has better upper stopband response compared to the other filters except the structures reported in [9, 10]. The proposed filter exhibits a compact size compared to the filters presented in [7, 8, 13, 14]. Shape factor of 0.86 can be witnessed for the proposed filter where the value is better than the filter reported in [9–11] and [13].

### Conclusion
In this paper, a novel Pythagorean fractal-based UWB BPF is proposed and verified successfully. It is the first time when the

| Ref. | Substrate: dielectric constant/height(mm) | \( |S_{11}| \) (dB) | \( |S_{21}| \) (dB) | Stopband (GHz) | Size (\( \lambda_g \)) at 6.85 GHz | S.F. \((M_{3dB}/M_{30dB})\) | 3 dB FBW (%) |
|------|----------------------------------------|------------------|------------------|----------------|-------------------------------|------------------|-------------|
| [7]  | 2.2/0.508                              | < -18            | > -1.1           | 14             | (0.99 × 0.53)                 | 0.9              | 126         |
| [8]  | 2.5/0.8                                | < -10            | > -2             | 17.1           | (0.77 × 0.47)                 | 0.921            | 117         |
| [9]  | 2.55/0.8                               | < -10            | > -1.5           | 18             | (0.66 × 0.21)                 | 0.8              | 122         |
| [10] | 2.2/0.508                              | < -10.5          | > -1             | 18             | (0.69 × 0.18)                 | 0.78             | 111         |
| [11] | 2.2/0.508                              | < -12            | > -1             | 17             | (0.645 × 0.318)               | 0.8              | 109.5       |
| [13] | 2.2/0.508                              | < -12            | > -0.8           | 16.6           | (0.53 × 0.44)                 | 0.8              | 110         |
| [14] | 10.2/1.28                              | < -15            | > -1             | 14             | (0.62 × 0.54)                 | 0.9              | 121         |
| This work | 4.4/1                                   | < -10            | > -1.5           | 17.5           | (0.52 × 0.43)                 | 0.86             | 103.9       |

S.F.-shape factor; FBW-fractional bandwidth.
Pythagorean tree is used as a stub to design the UWB filter. The benefit of multiple mode resonators is accomplished using the proposed resonator. For miniaturization purpose, the proposed fractal tree is used effectively to accomplish size reduction. The overall size of the structure is \((0.52 \lambda_d \times 0.43 \lambda_d)\). The selectivity is improved due to the presence of two TZs near the passband. The proposed filter has a compact size, excellent in-band and out-of-band characteristics with stopband extended up to 17.5 GHz which makes the filter suitable for any UWB systems.

References


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